

**II B. Tech I Semester Regular Examinations, March – 2014**  
**PROBABILITY THEORY AND STOCHASTIC PROCESSES**  
 (Electronics and Communications Engineering)

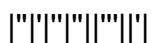
Time: 3 hours

Max. Marks: 75

Answer any **FIVE** Questions  
 All Questions carry **Equal** Marks  
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1. a) Give the definition and Axioms of probability.  
 b) Using Venn diagrams prove the Demorgan's laws:  
 i)  $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$       ii)  $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$       (7M+8M)
  
2. a) If the function  $G_x(x) = K \sum_{n=1}^N n^3 u(x-n)$  to be a valid distribution function, find the value of 'K'.  
 b) State and prove any four properties of probability density function.      (7M+8M)
  
3. a) Find the skew for Gaussian distributed random variable.  
 b) Explain about the monotonic transformations for a continuous random variable.      (7M+8M)
  
4. a) State and prove central limit theorem for equal distributions.  
 b) The joint density function of random variables X and Y is  

$$f_{xy}(x, y) = \frac{1}{a} e^{-|x|-|y|}, \quad -\infty < x < \infty, \quad -\infty < y < \infty.$$
 i) Are X and Y statistically independent variables.  
 ii) Calculate the probability of  $x \leq 1$  and  $y \leq 0$ .      (7M+8M)



5. a) Three random variables  $X_1$ ,  $X_2$ , and  $X_3$  represent samples of random noise voltage taken at three times. Their covariance matrix is defined by

$$[C_x] = \begin{bmatrix} 3.0 & 1.8 & 1.1 \\ 1.8 & 3.0 & 1.8 \\ 1.1 & 1.8 & 3.0 \end{bmatrix}$$

The transformation matrix

$$[T] = \begin{bmatrix} 4 & -1 & -2 \\ 2 & 2 & 1 \\ -3 & -1 & 3 \end{bmatrix}$$

Convert the variable to new random variables  $Y_1$ ,  $Y_2$  and  $Y_3$ . Find the covariance matrix of the new random variables.

- b) State and prove any two properties of joint characteristic function. (8M+7M)
6. a) Consider a random process  $X(t) = \cos(\omega t + \theta)$  where  $\omega$  is a real constant and  $\theta$  is a uniform random variable in  $(0, \frac{\pi}{2})$ . Find the average power in the process.
- b) Derive the condition for a random process to be mean Ergodic. (8M+7M)
7. a) State and prove any three properties of Cross correlation function.
- b) Derive the relation between Auto Correlation Function and PSD. (7M+8M)
8. a) Derive the relation between PSD of input & Cross PSD of input and output.
- b) A WSS process  $X(t)$  has  $R_{xx}(\tau) = Ae^{-a|\tau|}$  where  $A$  and 'a' are real constants is applied to input of LTI system with  $h(t) = e^{-bt} u(t)$ , where 'b' is a real positive constant. Find the PSD of the output of system. (7M+8M)



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1. a) State the following and explain
  - i) Baye's Theorem
  - ii) Conditional probability.
 b) What is the probability of picking an ace and a king from a 52 card deck? (9M+6M)
  
2. a) For a real constant  $b>0$ ,  $c>0$  and any 'a' find the condition on constant 'a' such that
 
$$f_x(x) = \begin{cases} 1 - \frac{x}{b}, & 0 \leq x \leq c \\ 0 & \text{elsewhere} \end{cases}$$
 is a valid pdf.
   
 b) State and explain the properties of conditional density function. (8M+7M)
  
3. a) Find the mean and variance of 'X + a', in terms of mean and variance of 'X'.
   
 b) Derive the relation between moment generating function and moments. (7M+8M)
  
4. a) Let X and Y are two independent random variables with
 
$$f_x(x) = \alpha e^{-\beta x} u(x)$$
 and
 
$$f_y(y) = \beta e^{-\beta y} u(y)$$
 Find the density function of  $Z = X + Y$  for i)  $\alpha \neq \beta$  ii)  $\alpha = \beta$ 
  
 b) Write the properties of Joint distribution. (8M+7M)
  
5. Zero mean Gaussian random variables  $X_1$ ,  $X_2$  and  $X_3$  having covariance matrix.
 
$$[C_x] = \begin{bmatrix} 4 & 2.05 & 1.05 \\ 2.05 & 4 & 2.05 \\ 1.05 & 2.05 & 4 \end{bmatrix}$$
 Are transformed to new random variable  $Y_1$ ,  $Y_2$ ,  $Y_3$ .
  - i) Find the covariance matrix of  $Y_1$ ,  $Y_2$  and  $Y_3$ .
  - ii) Write expression for joint density function of  $Y_1$ ,  $Y_2$  and  $Y_3$ . (15 M)



6. a) A random process  $X(t) = A \cos(\omega_c t + \theta)$  where  $\theta$  is a random variable uniformly distributed in the range  $(0, 2\pi)$ . Show that the process is ergodic in mean and correlation sense.
- b) Define covariance function and explain its properties. (8M+7M)
7. a) If the Auto Correlation Function of a WSS process is  $R(\tau) = Ke^{-k|\tau|}$ . Find its PSD.
- b) Check whether the following functions are valid PSDS or not. (8M+7M)
- i)  $\frac{w^2}{w^6 + 3w^2 + 3}$                       ii)  $\frac{w^2}{w^2 + 16}$
8. a) Compute the overall Noise figure of a four stage cascaded system with following data:  
 $F_1 = 10, F_2 = 5, F_3 = 8, F_4 = 12$   
 $ga_1 = 50, ga_2 = 20$  and  $ga_3 = 10$ .
- b) State and prove any three properties of Narrow band Noise processes. (8M+7M)



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1. a) Using Venn diagram and proof, prove that  

$$P(A \cup B/C) = P(A/C) + P(B/C) - P(A \cap B/C).$$
 b) Define probability in terms of relative frequency.  
 c) Explain independent events. (7M+3M+5M)
  
2. a) A random variable X is Gaussian with mean  $m_x = 0$  and  $\sigma_x = 1$ .  
 i) What is the probability that  $|X| > 2$ .    ii) What is the probability that  $X > 2$ .  
 b) Draw the pdf of Rayleigh density function by giving its expression and find the value and X where it is maximum. (8M+7M)
  
3. a) Let X be a random variable which can take values 1, 2, 3 with probabilities  $\frac{1}{3}, \frac{1}{6}$  and  $\frac{1}{2}$  respectively. Find the 3<sup>rd</sup> moment about the mean.  
 b) If X is the number scored in a throw of a fair die, show that Chebyshev's inequality gives  $P\{|x-m| > 2.5\} < 0.4$ , where 'm' is mean of X, while actual probability is zero. (7M+8M)
  
4. a) The joint density function of three random variables X, Y and Z is  

$$f_{xyz}(x, y, z) = 24xy^2z^3, \quad 0 < x < 1, 0 < y < 1, 0 < z < 1$$

$$= 0, \quad \text{otherwise.}$$
 i) Find the marginal densities  $f_x(x)$ ,  $f_y(y)$  and  $f_z(z)$ . ii) Find  $P(X > 1/2, y < 2, z > 1/2)$   
 b) State and prove any four properties of joint density function. (8M+7M)



5. For the joint characteristic function

$$Q_{xy}(w_1, w_2) = \exp\left[-\frac{1}{2}\left[\sigma_x^2 w_1^2 + 2\rho\sigma_x\sigma_y w_1 w_2 + \sigma_y^2 w_2^2\right]\right]$$

Find the Marginal characteristic functions of X and Y. (15M)

6. a) Consider a random process  $X(t) = 10\cos(100t + \varphi)$  where  $\varphi$  is uniformly distributed random variable in the interval  $(-\pi, \pi)$ . Show that the process is correlation ergodic.

b) State and prove any four properties of Auto Correlation Function. (7M+8M)

7. a) Derive the relation between PSD of  $x(t)$  and PSD of  $\frac{dx(t)}{dt}$ .

b) For a random process  $X(t) = A\cos(wt + \theta) + B\sin wt$  where A and B are two uncorrelated random variables with zero mean and equal variances and w is a real constant. Find the ACF of X(t) and hence its PSD. (7M+8M)

8. a) Derive the relation between input and output ACF of an LTI system with impulse response  $h(t)$ .

b) An amplifier with  $g_a = 40$  dB and  $B_N = 20$  kHz is found to have  $T_0 = 10^0$  K. Find  $T_e$  and Noise figure. (8M+7M)



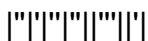
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1. a) State and prove Baye's Theorem.  
 b) If A and B are two mutually exclusive events show that
  - i)  $P(A/B) = \frac{P(A)}{1 - P(B)}$
  - ii)  $P(A/A \cup B) = \frac{P(A)}{P(A) + P(B)}$  if  $P(A \cup B) \neq 0$  (7M+8M)
  
2. a) Find the constant 'b' such that
 
$$f_x(x) = \begin{cases} \frac{e^{3x}}{4}, & 0 \leq x \leq b \\ 0, & \text{elsewhere} \end{cases}$$
 Is a valid density function.  
 b) State and prove any four properties of CDF. (7M+8M)
  
3. a) If X has density function
 
$$f_x(x) = \begin{cases} \exp(-x), & x > 0 \\ 0, & x \leq 0 \end{cases}$$
 Find the density function of  $Y = X^2$   
 b) Find the mean of a Gaussian distribution. (8M+7M)
  
4. a) State and prove the central limit theorem.  
 b) If X and Y are two Gaussian random variables with zero mean find the pdf of a new random variable  $Z = X+Y$ . (7M+8M)
  
5. a) State and explain the properties of jointly Gaussian random variables.  
 b) Random variables X and Y has joint density.
 
$$f_{xy}(x,y) = \frac{8}{3} u(x-2)u(y-1)xy^2 \exp(4-2xy)$$
 undergo a transformation
 
$$T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$
 to generate new random variables  $Y_1$  and  $Y_2$ . Find joint density of  $Y_1$  and  $Y_2$ . (7M+8M)



6. a) Consider a random process  $X(t) = A \cos wt$  where 'w' is a constant and A is uniformly distributed over (0, 1). Find the ACF and Auto covariance of X(t).  
 b) Explain mean Ergodic processes in brief. (8M+7M)
7. a) The PSD of a random process is  $S_{xx}(w) = \begin{cases} \pi, & |w| < 1 \\ 0, & \text{otherwise} \end{cases}$ . Find its ACF.  
 b) State and prove any three properties of Power Spectral Density. (8M+7M)
8. a) A random process X(t) has ACF  $R_{xx}(\tau) = A^2 + Be^{-|\tau|}$  where A, B are positive constants. Find the mean value of the system having impulse response  

$$h(t) = \begin{cases} e^{-wt}, & t > 0 \\ 0, & t < 0 \end{cases}$$
  
 b) Derive the equation for Noise figure of Cascaded system in terms of individual Noise figures (8M+7M)

